## A Proposed Experimental Procedure for the Vapor Pressure Osmometer to Minimize the Solute Dependence of the Calibration Constant

In two previous publications<sup>1,2</sup> the operating procedure of the Hewlett-Packard Model 302 Vapor Pressure Osmometer (VPO) was critically examined. One particularly disturbing result was the demonstration of a molecular weight dependence of the calibration constant. I now propose an approximate solution to this problem.

The theoretical model<sup>2</sup> developed to explain the solute dependence of the calibration constant shows that the temperature difference (and therefore resistance difference) between the two thermistors in the VPO can be represented by

$$\Delta R = \frac{KC_0}{M_n} \left(1 + \beta'/D_0\right) + \text{ higher order terms}$$
(1)

where K is the calibration constant as determined using a low-molecular-weight standard,  $C_0$  is the solution concentration,  $D_0$  is the average diffusion coefficient for the sample in the solvent at a given temperature, and  $\beta'$  is a constant for a given instrument configuration and solvent system.  $D_0$  can be shown<sup>3</sup> to be approximated by

$$D_0 = \Phi'(K'_s)^{1/3} / (M)^{1/2}$$
<sup>(2)</sup>

where  $\Phi$  is a universal constant,  $K_s$  the constant in the intrinsic viscosity-molecular weight relationship, and M is the molecular weight. While for a given temperature and solvent system  $K_s$  is variable for different molecular species, the variation of the cube root will be far less and consequently neglected as an approximation. Consequently, eq. (2) becomes

$$D_0 = \Phi/M^{1/2}$$
(3)

Ordinarily, for a polydisperse sample, M would be replaced by the viscosity-average molecular weight. However, the previous model shows that the diffusion of the smaller molecules produces the largest effects. Therefore, M is approximated by  $\overline{M}_n$  in eq. (3). Consequently, eq. (1) can be rewritten as

$$\Delta R = \frac{KC_0[1 + \beta(\overline{M}_n)^{1/2}]}{\overline{M}_n} + \text{ higher terms in } C_0$$
(4)

Equation (4) suggests a better calibration procedure so that the effects of the solute dependence of the calibration constant can be minimized. First, K can be determined in the usual manner using a low-molecular-weight standard by neglecting the second term in the brackets. Second,  $\beta$  is determined using a high-molecular-weight  $(\overline{M'_n} > 20,000)$  material of known molecular weight from

$$\beta = \frac{(S'\overline{M}'_n/K) - 1}{(\overline{M}'_n)^{1/2}}$$
(5)

where K is the calibration constant determined using the low-molecular-weight standard and S', the linear coefficient determined from a polynomial regression of  $\Delta R$  versus  $C_0$ .

From a solution of eq. (4), the number-average molecular weight  $(\overline{M}_n)$  of an unknown material can be shown to be

$$\overline{M}_n = \frac{K^2}{4S^2} \left[ \beta + \left( \beta^2 + \frac{4S}{K} \right)^{1/2} \right]^2 \tag{6}$$

where S is the linear coefficient in the polynomial regression of  $\Delta R$  versus  $C_0$ .

The application of the above method to polystyrene materials reported on previously<sup>1</sup> is given in Table I. The term  $\beta$  was determined by assuming  $\overline{M}_n = 21,500$  to be the actual value for this sample. The data show that using a secondary standard produces dramatically better agreement with the accepted values.

For high-molecular-weight materials, the use of two calibration constants has been demonstrated to yield  $\overline{M}_n$  values that are more consistent with those obtainable using the membrane osmometer. However, it may not be clear why a high-molecular-weight standard alone cannot be used for cali-

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Accepted $\overline{M}_n^{a}$	Hewlett-Packard Procedure <sup>b</sup>	Amoco <sup>c</sup>	Amocod
3525	3120	3500	3290
10,300	9480	10,400	10,400
20,000	18,850	21,500	21,500
51,000	38,000	49,000	46,000

TABLE I Molecular Weights of Standards by VPO

<sup>a</sup> Polystyrene determined by ArRo Labs.

<sup>b</sup> Run as suggested in Hewlett-Packard Model 302 VPO.

<sup>c</sup> Measured so as to eliminate drop size (Ref. 1).

<sup>d</sup> Calculated using the 21,500 sample as the high-molecular-weight standard as suggested in this publication.

bration. This can be answered in the following way: just as a calibration constant obtained with a low-molecular-weight standard gives incorrect values for  $\overline{M}_n$  when applied to high-molecular-weight polymers, a calibration constant obtained with a high-molecular-weight standard gives increasingly erroneous results as the  $\overline{M}_n$  of the unknown decreases.

## References

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